30th Postgraduate Combinatorial Conference

15th – 17th April 2026

University of Warwick

Conference programme

Invited Talks

Isoperimetric principles in combinatorics and analysis

Presenter: Keith Ball — University of Warwick
Date: Wednesday 15th April, **Time:** 10:40

Isoperimetric or deviation principles play an important role in analysis, geometry, combinatorics and probability. In this talk I will start by discussing the classical isoperimetric principle in Euclidean space, the Brunn-Minkowski inequality which implies it and an isoperimetric inequality in the ball which illustrates the power of the principle. I will then describe how expander graphs exhibit the same kind of behaviour, and provide a link with spectral gaps for Laplacian operators.

Lopisded Local Lemma in the setting of random injections

Presenter: Jan Volec — Czech Technical University in Prague
Date: Thursday 16th April, **Time:** 9:00

The lopsided Lovász local lemma due to Erdős and Spencer is a variant of the local lemma, where instead of independence one can limit so-called negative dependencies. In 2007, Lu and Székely described an easy-to-check condition for negative dependencies in the setting of uniformly random injections. In this talk, we present the Lu-Székely framework, and demonstrate it on various problems in extremal combinatorics.

Parts of this talk are based on joint works with Nina Kamčev and Benny Sudakov.

A bipartite version of the Bollobás–Eldridge–Catlin conjecture

Presenter: Julia Böttcher — London School of Economics
Date: Thursday 16th April, **Time:** 13:50

Bollobás and Eldridge, and independently Catlin conjectured that any n-vertex graph G with minimum degree $(dn-1)/(d+1)$ contains any n-vertex graph H with maximum degree at most d as a subgraph. This is open in general, but known for special cases, in particular when n is large, d is constant, and H is bipartite. In joint work with Peter Allen, Jozef Skokan, and Benny Sudakov, we show that for bipartite H a much better (and tight) minimum degree condition is possible. In the talk I will survey what is known on this conjecture, mention related problems and results, and outline how we obtain the new result combining probabilistic arguments with suitable constructions of expanders.

Applications and extensions of the Rödl Nibble

Presenter: Candida Bowtell — University of Birmingham
Date: Friday 17th April, **Time:** 9:00

The 'Rödl nibble' or 'semi-random method' was introduced by Rödl in 1985 to prove a conjecture of Erdős and Hanani concerning approximate designs. Pippenger generalised Rödl's result into a form which states that any approximately regular hypergraph with small codegrees has an almost-perfect matching. Since then, there have been many further developments. This talk will highlight some of the more recent extensions and their applications to problems concerning rainbow matchings in properly edge-coloured graphs.

Schedule: Wednesday 15th April 2026

Time	Information
10:00	*Registration*
10:30	*Welcome and general annoucements*
10:40	**Invited talk: Isoperimetric principles in combinatorics and analysis** Keith Ball — University of Warwick
11:30	**The number of occurrences of the two smallest distances** Cameron Strachan — London School of Economics
11:55	**On the Parameterized Complexity of Min-Sum-Radii** Pankaj Kumar — University of Birmingham
12:20	*Lunch Break*
13:50	**On the proportion of derangements in affine classical groups** Jessica Anzanello — University of Milano-Bicocca
14:15	**Monochromatic Sumsets in Countably Coloured Abelian Groups** Kada Williams — University of Cambridge
14:40	**Quasi-isometries, contractions, and intersection graphs** Chiara Molinari — University of Warwick
15:05	*Coffee break*
15:30	**Uniformly balanced H-factors in multicoloured complete graphs** Agnijo Banerjee — University of Cambridge
15:55	**Almost colour-balanced subgraphs – part 1** Emma Hogan — University of Oxford
16:20	**Almost colour-balanced subgraphs – part 2** Dmitry Tsarev — University of Oxford
17:40	**Film Night**

Schedule: Thursday 16th April 2026

Time	Information
09:00	**Invited talk: Lopisded Local Lemma in the setting of random injections** Jan Volec — Czech Technical University in Prague
09:50	**Exact co-degree bounds for tight Hamilton cycles** Arjun Ranganathan — University College London
10:15	**Spanning tight components in 4-uniform hypergraphs** Lu-Ming Zhang — London School of Economics
10:40	*Coffee break*
11:05	**Bounding the Lengths of Minimal B-Saturated Words over a Transitive Alphabet** Zhongyi (Pierre) Zhou — University of St Andrews
11:30	**Automatic discovery of generating trees for pattern-avoiding inversion sequences** Abigail Ollson — Keele University
11:55	**Properties of the Wiener index** Isabella Fosberry — Lancaster University
12:20	*Lunch Break*
13:50	**Invited talk: A bipartite version of the Bollobás–Eldridge–Catlin conjecture** Julia Böttcher — London School of Economics
14:40	**Distribution of independent sets in perfect r-ary trees** Daniel Il'kovič — University of Leipzig / University of Warwick
15:05	**Reconstructing a graph from its Bell colouring graph** Brian Hearn — London School of Economics
15:30	*Coffee break*
15:55	**Approximate sampling from the random cluster model via Markov chains** Paulina Smolárová — University of Oxford
16:20	**Cycles in Brain Networks: Mapping Feedback Architecture in the *C. elegans* Connectome** Karim Bibars — Nottingham Trent University
16:45	**Majority Dynamics on Graphs** Alan Sergeev — University of Birmingham
18:00	**Conference Dinner**

Schedule: Friday 17th April 2026

Time	Information
09:00	**Invited talk: Applications and extensions of the Rödl Nibble** Candida Bowtell — University of Birmingham
09:50	**Embeddings of Tree-Like Structures in Directed Graphs** Anna Monger — University of Birmingham
10:15	**On the maximum and negative frustration indices of graphs** Huiying Zeng — Durham University
10:40	*Coffee break*
11:05	**Sparse Partitions of Graphs with Bounded Clique Number** Toby Insley — University College London
11:30	**Cycle-factors of regular graphs via entropy** Lukas Michel — University of Oxford
11:55	**Sparse Counting Lemma for K_4** Warach Veeranonchai — Royal Holloway, University of London
12:20	*Lunch Break*
13:50	**Temporal Role Colouring** Ella Yates — University of Glasgow
14:15	**A new FPT result for Temporal Dominating Set** Elena Moss — University of Glasgow
14:40	*Coffee break*
15:05	**Multicolour size-Ramsey number for paths** Csongor Beke — University of Cambridge
15:30	**Zero Sum Ramsey numbers are linear on d-degenerate Graphs** Andrey Shapiro — Kings College London
15:55	**Decay-Ramsey Sequences** Pedram Noohi — London School of Economics

Talk Abstracts

Abstracts are listed in order of presentation.

The number of occurrences of the two smallest distances

Presenter: Cameron Strachan – London School of Economics
Date: Wednesday 15th April **Time:** 11:30

For each value of $\rho > 1$, let $e(n, \rho)$ denote the maximum combined number of occurrences of the smallest and second smallest distances in a set of n points in the plane, given that the smallest distance is 1 and the second smallest distance is ρ. Extending work of Vesztergombi and Csizmadia, we determine the asymptotics of $e(n, \rho) = f(\rho)n - o(n)$ for all except six values of ρ. For these exceptional values, we determine upper and lower bounds that are quite close. The hardest exceptional value is the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$, where we give the bounds $53/13 \leq f(\varphi) \leq 141/34$, with the lower bound disproving a conjecture of Csizmadia. It turns out that for all except finitely many values of ρ, $f(\rho) \in \{3, 7/2\}$. For $\rho = \varphi$, we find the upper bound by solving a large LP. For the other cases, we redraw the graph of the two smallest distances as a planar graph or a graph closely related to a planar graph.

On the Parameterized Complexity of Min-Sum-Radii

Presenter: Pankaj Kumar – University of Birmingham
Date: Wednesday 15th April **Time:** 11:55

In the MIN-SUM-RADII (MSR) clustering problem, we are given a finite set X of n points in a metric space. The objective is to find at most k clusters centered at a subset of these points such that every point of X is assigned to one of the clusters, minimizing the sum of the radii of the clusters. The problem is known to be NP-hard even on metrics induced by weighted planar graphs and metrics with constant doubling dimension, as shown by Gibson et al. (SWAT 2008). In this work, we investigate the parameterized complexity of MSR on metrics induced by undirected graphs. We distinguish between weighted graph metrics (with positive edge weights) and unweighted graph metrics (where all edges have unit weight).

Weighted Graph Metrics. We show that MSR is W[1]-hard on metrics induced by weighted bipartite graphs, when parameterized by the combined parameter k the number of clusters and Delta the cost of the clustering. We then investigate the structural parameterized complexity of the problem. Drexler et al. [doi:10.48550/arXiv.2310.02130] showed that the MSR problem admits an XP algorithm on metrics induced by weighted graphs when parameterized by treewidth, and asked whether this can be improved to fixed-parameter tractability. We first answer their question in the negative, and more strongly show that MSR stays W[1]-hard on metrics induced by undirected weighted bipartite graphs when parameterized by the vertex cover number plus k. We then turn our attention to parameters for dense graphs and show that MSR remains W[1]-hard when parameterized by k+Delta even on cliques and complete bipartite graphs.

On the positive side, we employ the known XP algorithm parameterized by treewidth, to show that the MSR problem is FPT when parameterized by the parameter treewidth plus Delta. Together, these results provide a complete picture of the parameterized complexity of MSR with respect to any combination of parameters k, Delta, as well as structural parameters for sparse graphs above vertex cover and known parameters for dense graphs (such as neighborhood diversity and modular width).

Unweighted Graph Metrics. The story is rather different for unweighted graphs, since it is a long standing open question whether MSR on metrics induced by undirected graphs is solvable in polynomial-time. Although we cannot answer this question, we provide classical and parameterized hardness results for two very closely related problems, namely EXACT-MSR (MSR and one wants to find exactly k clusters) and ALLOWED-CENTERS(MSR with an additional set of allowed cluster centers). We also show that MSR as well as these two problems are fixed-parameter tractable parameterized by the treedepth of the input graph.

On the proportion of derangements in affine classical groups

Presenter: Jessica Anzanello – University of Milano-Bicocca
Date: Wednesday 15th April **Time:** 13:50

A derangement is a permutation without fixed points. By a classical theorem of Jordan, every nontrivial finite transitive permutation group contains such elements. This fundamental fact has far-reaching applications and raises naturally questions about the abundance and orders of derangements, which have been widely studied in recent years. In this talk, I will discuss how combinatorial tools, including cycle indices, generating functions for partitions satisfying suitable fixed-point conditions, and q-polynomial identities, can be used to obtain exact and simple formulas for the proportion of derangements in the finite affine general linear, unitary, symplectic and orthogonal groups.

Preprint: arXiv:2508.07093, to appear in Forum Math. Sigma.

Monochromatic Sumsets in Countably Coloured Abelian Groups

Presenter: Kada Williams – University of Cambridge
Date: Wednesday 15th April **Time:** 14:15

Fernández-Bretón, Sarmiento, and Vera [1] showed that whenever a direct sum of sufficiently many (omega iterates of the power set of omega) copies of the cyclic group of order 4 is countably coloured, there exist arbitrarily large finite sets whose sumsets are monochromatic. They asked whether every abelian group that has no element of order 4 can be countably coloured such that there is not even a monochromatic sumset $\{2x, x + y, 2y\}$ of a set $\{x, y\}$ with two elements. We answer this question affirmatively.

Published: I. Leader and K. Williams, Monochromatic sumsets in countable colourings of abelian groups, Bull. Pol. Acad. Sci. Math. **72** (2024), no. 2, 97–102; MR4935864

References

[1] David J. Fernández-Bretón, Eliseo Sarmiento Rosales, and Germán Vera. "Owings-like theorems for infinitely many colours or finite monochromatic sets". In: *Ann. Pure Appl. Logic* 175.10 (2024), Paper No. 103495, 9. ISSN: 0168-0072,1873-2461.

Quasi-isometries, contractions, and intersection graphs

Presenter: Chiara Molinari – University of Warwick
Date: Wednesday 15th April **Time:** 14:40

Coarse graph theory studies graphs from a geometric perspective, focusing on their large-scale properties, in particular those that are preserved under quasi-isometry (a generalization of a bi-Lipschitz map between metric spaces that allows for an additive error). Indeed, graphs can be naturally seen as metric spaces, if equipped with the natural graph distance.

In this talk, we consider classes of graphs with topological properties, such as trees and planar graphs, and investigate how certain operations (edge contractions, tree decompositions, construction of iterated intersection graphs) affect their quasi-isometry classes.

We will present a characterization of graphs that are quasi-isometric to planar graphs (quasi-planar) as *iterated cover-intersection graphs with subdivisions* of planar graphs. We will also discuss related results. For example, the class of quasi-planar graphs is closed under edge contractions, in analogy to the case of planar graphs, where contracting infinitely many edges does not break planarity. In a similar flavor, given a graph that admits a tree decomposition with adhesion sets of bounded diameter, it is sufficient for each bag to be quasi-planar (with uniform constants) to achieve quasi-planarity of the whole graph.

Uniformly balanced H-factors in multicoloured complete graphs

Presenter: Agnijo Banerjee – University of Cambridge
Date: Wednesday 15th April **Time:** 15:30

A *balanced colouring* of a graph is one in which every colour appears the same number of times. Given a fixed graph H on r vertices and a balanced k-colouring of the complete graph K_{nrk}, Hollom (2025) asked the following question: can we always find an H-factor F covering all vertices of the complete graph K_{nrk} such that the inherited colouring of F is almost balanced? This is known to be the case for palettes of only two colours, or when H is only a single edge. We answer the above question in full, finding an H-factor which is at most $C_{r,k}$ edges away from being balanced, where $C_{r,k}$ depends only on r and k. In fact, we work in the more general setting wherein our palette of colours is a subset of $\mathbb{S}^{d-1}$, and find an H-factor where the sum of the colours of all edges has bounded Euclidean norm.

Preprint: arXiv:2601.18789

Almost colour-balanced subgraphs (two-part talk)

Presenters: Emma Hogan and Dmitry Tsarev – University of Oxford
Date: Wednesday 15th April **Time:** 15:55

Let G be a graph, and let $c : E(G) \to [k]$ be a k-edge-colouring of G. We call c colour-balanced if every colour class has the same size. Inspired by zero-sum Ramsey problems, Kittipassorn and Sinsap asked whether every colour-balanced k-edge-colouring of the complete graph K_{2kt} admits a colour-balanced perfect matching M. While this is true when $k = 2$, Pardey and Rautenbach give a counterexample for $k = 3$, showing that a 3-edge-coloured K_6 need not admit a colour-balanced matching. Letting $f(M) = \sum_{i=1}^{k} ||c^{-1} \cap M| - t|$, they instead conjecture that every colour-balanced k-edge-colouring of K_{2kt} admits a perfect matching M satisfying $f(M) = O(k^2)$. We prove this conjecture, improving the previous best-known bound of $f(M) \leq 4^{k^2}$ due to Hollom. In fact, we obtain analogous bounds in a far more general setting, where edges of a complete graph are labelled with vectors in $\mathbb{R}^k$ instead of colours, and we are searching for a 'representative' copy of any given bounded-degree spanning subgraph. We also obtain similar bounds for matchings in complete r-uniform hypergraphs. This answers a question recently posed by Banerjee and Hollom, and significantly improves earlier bounds for several classes of subgraphs.

This is based on our joint work with Alex Scott and Youri Tamitegama.

Exact co-degree bounds for tight Hamilton cycles

Presenter: Arjun Ranganathan – University College London
Date: Thursday 16th April **Time:** 09:50

Over recent decades, finding hypergraph extensions of Dirac's theorem [1] – that every n-vertex graph with minimum degree at least $n/2$ contains a Hamilton cycle – has become an extensive research theme in extremal graph theory. These questions were first addressed by Katona and Kierstead [2], who conjectured that, for all uniformities $k \geq 3$, any n-vertex k-graph G having the property that every $(k-1)$-set extends to at least $\frac{n-k+2}{2}$ edges contains a *tight Hamilton cycle* (a cyclic ordering of the vertices such that every set of k consecutive vertices forms an edge). Rödl, Ruciński and Szemerédi [4, 5] first proved this conjecture asymptotically, showing that $n/2 + o(n)$ suffices, and later obtained an exact bound for $k = 3$ [6].

Our main result is a complete resolution of this conjecture for all uniformities $k \geq 3$, improving the above asymptotic bounds to exact ones. In this talk, we will present some of the key ideas behind our proof. While we will largely describe our analysis for near-extremal hypergraphs, we will briefly mention our strategy in the non-extremal regime, where we use a recent Hamiltonicity framework of Lang and Sanhueza-Matamala [3] and avoid traditional approaches using regularity.

This talk is based on joint work with Richard Lang, Shoham Letzter and Nicolás Sanhueza-Matamala.

References

[1] G. A. Dirac. "Some theorems on abstract graphs". In: *Proc. Lond. Math. Soc.* 3.1 (1952), pp. 69–81.

[2] G. Y. Katona and H. A. Kierstead. "Hamiltonian chains in hypergraphs". In: *J. Graph Theory* 30.3 (1999), pp. 205–212.

[3] R. Lang and N. Sanhueza-Matamala. "A hypergraph bandwidth theorem". In: *arXiv:2412.14891* (2024).

[4] V. Rödl, A. Ruciński, and E. Szemerédi. "A Dirac-type theorem for 3-uniform hypergraphs". In: *Combin. Probab. Comput.* 15.1-2 (2006), pp. 229–251.

[5] V. Rödl, A. Ruciński, and E. Szemerédi. "An approximate Dirac-type theorem for k-uniform hypergraphs". In: *Combinatorica* 28.2 (2008), pp. 229–260.

[6] V. Rödl, A. Ruciński, and E. Szemerédi. "Dirac-type conditions for Hamiltonian paths and cycles in 3-uniform hypergraphs". In: *Adv. Math.* 227.3 (2011), pp. 1225–1299.

Spanning tight components in 4-uniform hypergraphs

Presenter: Lu-Ming Zhang – London School of Economics
Date: Thursday 16th April **Time:** 10:15

We prove that every n-vertex 4-uniform hypergraph with minimum codegree at least $\lfloor n/4 \rfloor$ has a spanning tight component. This is tight, and it settles the 4-uniform case of a conjecture of Illingworth, Lang, Müyesser, Parczyk, and Sgueglia.

Joint work with Francesco Di Braccio, Brian Hearn, Joanna Lada, and Mihir Neve.

Bounding the Lengths of Minimal B-Saturated Words over a Transitive Alphabet

Presenter: Zhongyi (Pierre) Zhou – University of St Andrews
Date: Thursday 16th April **Time:** 11:05

Let $\mathcal{A}$ be a finite set of letters, and $I \subseteq \mathcal{A} \times \mathcal{A}$ an independence relation. We say that $(\mathcal{A}, I)$ is a *transitive alphabet* if the independence relation I is transitive.

Let $(\mathcal{A}, I)$ be a transitive alphabet, and $B \subseteq \mathcal{A}^*$ a finite antichain. A word $w \in \mathcal{A}^*$ is said to be B*–saturated* if for all $v \in [w]$, there exists some $\beta \in B$, such that $\beta \leq v$. By Higman's Lemma, the set $W_{\text{sat}}(B)$ of all minimal B-saturated words over A is finite.

The aim of this talk is to demonstrate that there exists a computable bound for the lengths of minimal B-saturated words over a transitive alphabet $(\mathcal{A}, I)$ whose graph realization $\Gamma(\mathcal{A}, I)$ is connected. This means that there exists an algorithm that, given a transitive alphabet $(\mathcal{A}, I)$ of this kind, computes the set $W_{\text{sat}}(B)$. If time allows, I will also briefly discuss the case in which the transitive alphabet $(\mathcal{A}, I)$ becomes disconnected, and explain the motivations behind these problems.

Automatic discovery of generating trees for pattern-avoiding inversion sequences

Presenter: Abigail Ollson – Keele University
Date: Thursday 16th April **Time:** 11:30

A size n *inversion sequence* is a word $w_1 w_2 ... w_n \in \mathbb{N}^*$ such that any value w_i at index i satisfies $w_i < i$. For example, 0023, 00031, and 01234 are all inversion sequences. However, 004 is not an inversion sequence as $4 \geq 3$. Inversion sequences are in bijection with permutations, so there are $n!$ inversion sequences of size n. An inversion sequence σ can be represented on a graph with points at (i, σ_i) for $i \in \{1, 2, \ldots, n\}$. For example, the inversion sequence 01031 is shown in 1. Due to the restriction on values in an inversion sequence, all of the points in an inversion sequence must be below the dashed line in Figure 1.

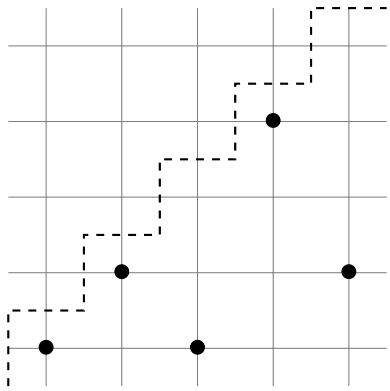

Figure 1: The inversion sequence 01031.

We define a *pattern* to be a word $\pi \in \mathbb{N}^*$ which contains at least one occurrence of every value between 0 and the maximum value in π. patterns are not necessarily inversion sequences, for example 3210 and 1010 are patterns but not inversion sequences and 0002 is an inversion sequence but not a pattern.

An inversion sequence σ *contains* a pattern π if there is a subsequence of σ, not necessarily at consecutive indices, which is order isomorphic to π. For example, in the inversion sequence 01031123 the subsequence 1323 implies an occurrence of the Cayley permutation 0212. If an inversion sequence does not contain a pattern we say it *avoids* it, so 0121 avoids both 210 and 000 for example. An inversion sequence avoids a set of patterns if it avoids every pattern in the set, otherwise it contains the set.

For a set of patterns B, we define the set of inversion sequences that avoid all patterns in B as $I(B)$. We call $I(B)$, or simply I, an *inversion class* and B a *basis* of the inversion class. For example, the inversion class $I(010)$ is all inversion sequences which do not contain the pattern 010. The inversion sequences 0000, 0111, 0122 and 0123 are in $I(010)$ but 0010, 0110, 0020 and 0121 are not. A basis for an inversion class is not unique, for example the inversion sequences in $I(010, 0021)$ are the same as those in $I(10)$.

Our aim is to enumerate inversion classes by finding a generating function for the class. This has been completed for all classes avoiding one or two patterns of size 3, see Testart [1]. A common method in literature of enumerating inversion classes, and one which we will use in this paper, is *generating trees*. A generating tree is a rooted, labelled tree such that the label of each node determines the labels of the children of that node. In particular, a *combinatorial generating tree* is a generating tree such that each label of a node is an object in the combinatorial class. Therefore, the combinatorial generating tree for an inversion class $I(B)$ is a rooted tree with each inversion sequence of size n in $I(B)$ the label of a node at level n. One way to create such a tree is to add new rightmost values to inversion sequences, called a generating tree *growing on the right*.

In the literature, to enumerate an inversion class instead of using inversion sequences often new labels are created which determine the children of each node. By using as few labels as possible this creates a *concise* tree, a generating tree for which no two subtrees are isomorphic and have different labels at their roots. A concise generating tree is isomorphic to the combinatorial generating tree for the inversion class. Our method finds concise generating trees which are growing on the right.

We present a method which is similar to Testart [1, Chapter 2] as both methods consider the prefixes of inversion sequences. However, we also make commitments about which of the values already seen in the prefix will appear again in the inversion sequences in the children of a node and which will not. Using our method, we have written code to automatically discover concise generating trees for inversion classes. We have also proven conditions for when an inversion class can be represented by a finite generating tree in this way.

References

[1] Benjamin Testart. "Completing the enumeration of inversion sequences avoiding one or two patterns of length 3". English. In: *Electron. J. Comb.* 32.4 (2025), research paper p4.46, 70. ISSN: 1077-8926.

Properties of the Wiener index

Presenter: Isabella Fosberry – Lancaster University
Date: Thursday 16th April **Time:** 11:55

The Wiener index $W(G)$ of a graph G is a graph invariant defined on connected graphs and takes integer values, and is related to boiling points of alkanes in chemistry. In this talk, we define this invariant and review some of its properties as well as its relationship to the Szeged index $Sz(G)$, another graph invariant. We then consider related results for variable versions of the Wiener index and Szeged index. Finally we describe some open problems about transmission irregular trees.

Distribution of independent sets in perfect r-ary trees

Presenter: Daniel Iľkovič – University of Leipzig / University of Warwick
Date: Thursday 16th April **Time:** 14:40

Motivated by a generalisation of the Erdős-Ko-Rado Theorem to the setting of independent sets in graphs, Hurlbert and Kamat conjectured that for every tree T and every k, the maximum of $|\mathcal{I}_T^k(v)|$ can always be attained by a leaf of T. While this conjecture turns out to be false in general, it is known to hold for specific families of trees, and this talk will prove that this conjecture holds for a new family of trees, the perfect r-ary trees.

Reconstructing a graph from its Bell colouring graph

Presenter: Brian Hearn – London School of Economics
Date: Thursday 16th April **Time:** 15:05

The *Bell colouring graph* $\mathcal{B}(G)$ of a graph G is the graph whose vertices are the partitions of the vertex set of G into independent sets, with an edge between two partitions if and only if one can be obtained from the other by changing the part of a single vertex of G. Given a natural number k, the *Bell k-colouring graph* $\mathcal{B}_k(G)$ and the *upper-Bell k-colouring graph* $\mathcal{B}_{\geq k}(G)$ are the induced subgraphs of $\mathcal{B}(G)$ consisting of all partitions with at most k parts and at least k parts, respectively.

We determine precisely when two finite graphs have isomorphic Bell colouring graphs. In particular, we show that every n-vertex graph G with no vertices of degree $n-1$ is uniquely determined by its Bell colouring graph $\mathcal{B}(G)$, and by its upper-Bell colouring graph $\mathcal{B}_{\geq k}(G)$ if $k < n-1$. We also show that every n-vertex graph with maximum degree $\Delta(G) < \frac{1}{9}n - \frac{1}{3}$ is uniquely determined by its Bell k-colouring graph $\mathcal{B}_k(G)$ if $k > \chi(G)$. By taking graph complements, each of these results can be restated in terms of partitions into cliques.

Approximate sampling from the random cluster model via Markov chains

Presenter: Paulina Smolárová – University of Oxford
Date: Thursday 16th April **Time:** 15:55

In many settings, we are interested in sampling from a complex distribution, where exact sampling is computationally hard. However, often we are nevertheless able to design an approximate sampler, whose distribution is "not far" from the desired distribution.

A common approach is to use Markov chains: one can obtain an approximate sampling algorithm by designing a Markov chain with the desired stationary distribution and proving that it converges in a polynomial number of steps. Most existing methods focus on the worst-case convergence, even though in algorithm design, we can choose the initial state, and thus potentially dramatically improve the convergence time.

In this talk, I will use an example of the random cluster model, a hard sampling problem generalizing independent percolation. I will show how to bound the convergence time for a simple Markov chain with a suitable initialisation to obtain a fast sampling algorithm on random graphs, including on Erdős-Rényi graphs, where previously no efficient algorithms were known.

Cycles in Brain Networks: Mapping Feedback Architecture in the *C. elegans* Connectome

Presenter: Karim Bibars – Nottingham Trent University
Date: Thursday 16th April **Time:** 16:20

Network science has become a central framework for studying brain connectomes, where graph-theoretic measures are routinely used to describe organisation and identify important nodes or subgraphs. Metrics such as degree, k-core structure, and rich-club organisation have revealed dense backbones and hierarchical organisation in neural networks, yet they primarily characterize connectivity patterns rather than the presence of feedback and recurrence. Directed recurrence is a fundamental prerequisite for internal network dynamics, but its organisation at the scale of entire connectomes remains poorly understood.

Here, we introduce a cycle-based framework to map the feedback architecture of *C. elegans*, the only organism with a near-complete neuronal wiring diagram. We quantify how nodes and subnetworks participate in directed cycles and overall connectivity and relate recurrence to biological organisation. To situate recurrence within the broader network architecture, we apply dimensionality reduction to uncover latent structure.

Preliminary analyses reveal a dissociation between hubness and recurrence, suggesting that connectivity alone does not fully capture the network's feedback architecture.

Majority Dynamics on Graphs

Presenter: Alan Sergeev – University of Birmingham
Date: Thursday 16th April **Time:** 16:45

Given a simple graph $G = (V, E)$ and a map $l_0 : V \to \{+1, -1\}$, the *majority dynamics* on G with *initial assignment* of states l_0 is a process that begins on day 0, and for each $t \geq 0$ produces a new assignment of states l_{t+1} where each vertex takes the state of the majority of its neighbours, and remains at its previous state in the case of a tie. Specifically, for each $v \in V$,

$$l_{t+1}(v) = \begin{cases} +1 & \text{if } \sum_{u \in N(v)} l_t(u) > 0, \text{ or } \sum_{u \in N(v)} l_t(u) = 0 \text{ and } l_t(v) = +1, \\ -1 & \text{otherwise.} \end{cases} \tag{0.1}$$

This process is a model for opinion exchange dynamics, with applications in many areas, such as politics, sociology, biophysics. While there exist results about the process eventually reaching a 2-periodic stable state on all graphs, a natural question to study would be under which initial conditions is unanimity reached after a finite number of steps.
When considering the question specifically in the case of Binomial Random Graphs, a longstanding conjecture is the following:

Conjecture 0.1. *Let $G \sim G(n, p)$ be the binomial random graph with $p = \omega(1/n)$ and $l_0(v)$ be sampled uniformly at random from $\{+1, -1\}$ for each $v \in V$. Then w.h.p. the majority dynamics process reaches unanimity after sufficiently many steps t.*

Steps towards proving the conjecture have been taken taken by gradually improving the range of p for which the conjecture is known to be true. It was first shown that unanimity is reached in four rounds for $p \geq cn^{-1/2}$ for c a constant depending on ε. The case for $C_\varepsilon n^{-2/3} \log^{2/3} n \leq p < cn^{-1/2}$ was proved later, but progress has stalled for smaller p. Our work aims to prove the conjecture for the range $n^{1/4} \ll p \leq \mathcal{O}(n^{1/3})$, and lays the groundwork for proving the conjecture in the general case $n^{1/(k+1)} \ll p \leq \mathcal{O}(n^{1/k})$.

Embeddings of Tree-Like Structures in Directed Graphs

Presenter: Anna Monger – University of Birmingham
Date: Friday 17th April **Time:** 09:50

In 1978, Bollobás conjectured that, for n sufficiently large, every graph on n vertices with minimum degree at least $(1/2 + o(1))n$ embeds every spanning tree with bounded maximum degree [1]. In 1995, Komlós, Sárközy and Szemerédi gave a proof of this conjecture [4], and in 2001 they extended this result to apply to trees with maximum degree up to $cn/\log n$ [3].

Recent work by Mycroft and Naia, and Kathapurkar and Montgomery extends these results in the setting of directed graphs [6, 2], and work by Montgomery, Müyesser and Pehova generalises the problem to transversal embeddings [5].

This talk will discuss these existing results, as well as related ongoing joint work with Candida Bowtell.

References

[1] Bela Bollobas. *Extremal graph theory.* English. Vol. 11. Lond. Math. Soc. Monogr. Academic Press, London, 1978.

[2] Amarja Kathapurkar and Richard Montgomery. “Spanning trees in dense directed graphs”. en. In: *Journal of Combinatorial Theory, Series B* 156 (Sept. 2022), pp. 223–249. ISSN: 00958956. (Visited on 03/16/2026).

[3] Janos Komlos, Gábor N. Sárközy, and Endre Szemerédi. “Spanning Trees in Dense Graphs”. In: *Combinatorics, Probability and Computing* 10 (Sept. 2001), pp. 397–416.

[4] János Komlós, Gábor N. Sárközy, and Endre Szemerédi. “Proof of a packing conjecture of Bollobás”. English. In: *Comb. Probab. Comput.* 4.3 (1995), pp. 241–255. ISSN: 0963-5483.

[5] Richard Montgomery, Alp Müyesser, and Yani Pehova. “Transversal factors and spanning trees”. en. In: *Advances in Combinatorics* (May 2022). (Visited on 03/16/2026).

[6] Richard Mycroft and Tássio Naia. “Trees and Treelike Structures in Dense Digraphs”. In: *Combinatorics, Probability and Computing* (Jan. 2026). ISSN: 0963-5483. (Visited on 03/16/2026).

On the maximum and negative frustration indices of graphs

Presenter: Huiying Zeng – Durham University
Date: Friday 17th April **Time:** 10:15

A signed graph is a graph with signatures ($+1$ or -1) on its edges. A cycle is called positive if the product of its edge signatures is positive, and a signed graph is called balanced if each cycle in it is positive. The frustration index is the minimum number of edges whose deletion makes the signed graph balanced, which is considered to be a measurement of the imbalance of the signed graph. In this paper, we compare the frustration index of the all-negative signature with the maximum frustration index of all possible signatures on the unsigned graph. We classify a few families of graphs that contain a dominating vertex into three scenarios: the all-negative signature does not reach the maximum frustration, the all-negative signature reaches the maximum frustration index non-uniquely, and the all-negative signature reaches the maximum frustration index uniquely. Moreover, for the families of fan graphs and wheel graphs, we characterise the signatures reaching the maximum frustration index.

Sparse Partitions of Graphs with Bounded Clique Number

Presenter: Toby Insley – University College London
Date: Friday 17th April **Time:** 11:05

Given a graph H, say that a graph G is *H-free* if G contains no induced copy of H. A number of conjectures, most notably one due to Erdős and Hajnal, claim that H-free graphs exhibit a highly atypical structure. A particularly strong conjecture in this direction was recently asked by Fox, Nguyen, Scott and Seymour. Given a fixed graph H, is there some constant $C = C(H) > 0$ such that for every $\varepsilon > 0$ and H-free graph G, there is a partition of $V(G)$ into at most $(1/\varepsilon)^C$ sets $A_1, \ldots, A_t$ such that either $G[A_i]$ or its complement graph has maximum degree at most $\varepsilon|A_i|$ for each $i \in \{1, \ldots, t\}$? The mentioned authors answered this in the positive for $H = P_4$, and asked whether the same can be said when H is a triangle. We answer their question, more generally showing that the above is true when H is any complete graph.

Joint work with António Girão.

Cycle-factors of regular graphs via entropy

Presenter: Lukas Michel – University of Oxford
Date: Friday 17th April **Time:** 11:30

It is a classical result that a random permutation of n elements has, on average, about $\log n$ cycles. We generalise this fact to all directed d-regular graphs on n vertices by showing that, on average, a random cycle-factor of such a graph has $\mathcal{O}((n \log d)/d)$ cycles.

This is tight up to the constant factor and improves the best previous bound of the form $\mathcal{O}(n/\sqrt{\log d})$ due to Vishnoi. It also yields randomised polynomial-time algorithms for finding such a cycle-factor and for finding a tour of length $(1 + \mathcal{O}((\log d)/d)) \cdot n$ if the graph is connected. The latter result makes progress on a restriction of the Traveling Salesman Problem to regular graphs, a problem studied by Vishnoi and by Feige, Ravi, and Singh. Our proof uses the language of entropy to exploit the fact that the upper and lower bounds on the number of perfect matchings in regular bipartite graphs are extremely close.

This talk is based on joint work with Micha Christoph, Nemanja Draganić, António Girão, Eoin Hurley, and Alp Müyesser.

Sparse Counting Lemma for K_4

Presenter: Warach Veeranonchai – Royal Holloway, University of London
Date: Friday 17th April **Time:** 11:55

The sparse analogue of Szemerédi's regularity method has played a central role in the development of extremal results for random graphs. While the sparse embedding lemma (the KLR conjecture) has been resolved, the corresponding sparse counting lemma remains widely open. The conjecture, formulated by Gerke, Marciniszyn, and Steger, states that for every fixed graph H and any $\beta > 0$, there exists $\varepsilon > 0$ such that the following holds. Consider a balanced blow-up of H with vertex classes of size n, where each pair corresponding to an edge of H forms an (ε)-regular bipartite graph with exactly m edges. Assume that m is above the natural threshold $m \gg n^{2-1/m_2(H)}$, then all but a β^m proportion of such graphs contain at least $(1 - \delta)$ times the expected number of copies of H.

At present, among the complete graphs, the conjecture is known only for $H = K_3$. In this paper, we establish the $H = K_4$ case of the conjecture.

Temporal Role Colouring

Presenter: Ella Yates – University of Glasgow
Date: Friday 17th April **Time:** 13:50

A *role colouring* of a graph G is an assignment of colours to the vertices of G such that two vertices of the same colour have identical sets of colours in their neighbourhoods. For a fixed graph R the R-role colouring (or assignment) problem asks whether G has a role colouring that is consistent with the adjacencies of R. More formally, it asks for a vertex mapping $r : V(G) \to V(R)$, such that $r(N_G(u)) = N_R(r(u))$ for all $u \in V(G)$. If such a mapping exists the graph G is called *R-role colourable*.

In this talk, we define an extension of the R-role colouring problem to *temporal graphs*, these are graphs in which the set of active edges changes over time. We define roles in the temporal sense via an automaton with states capturing both the current colour of a vertex and potentially some information about what other colours have been adjacent to this vertex while it has had its current colour. Transitions between these states are labelled with subsets of colours; informally, we allow a vertex v to move to a new state on timestep t along an arc labelled with the subset of colours C' if on timestep t the set of colours appearing on neighbours of v is precisely C'.

The R-role colouring problem is known to be NP-hard for connected role graphs with three or more vertices[1], we show that equivalent results hold in the temporal A-role colouring problem so the problem inherits hardness from the static case. To contend with the tractability problem we explore temporal parameters where this problem can be solved more efficiently.

This talk is based on joint work with Jess Enright, Kitty Meeks and Puck Rombach.

A new FPT result for Temporal Dominating Set

Presenter: Elena Moss – University of Glasgow
Date: Friday 17th April **Time:** 14:15

The Dominating Set problem on graphs allows us to model real-world scenarios that depend on interactions between objects, people, or places. However, often such relationships change over time — this temporal aspect is not covered by the classical definition of graphs. We study a version of Dominating Set on *temporal graphs*, which are graphs in conjunction with an edge labelling function, that denotes the times where an edge is "active". With the known NP-hardness of Dominating Set in the static case, we cannot hope to achieve efficient results in the classical sense; however, recent progress has been made in the *parametrised hardness* of the temporal problem. In this talk, we will answer (positively) an open question about the hardness of Temporal Dominating Set when parametrised by *temporal neighbourhood diversity* — giving us a finer parametrisation than previously known. Broadly, this parameter can be viewed as how many clusters of vertices have the same neighbours, at the same times. Finally, we will discuss other refinements of the parameter, and potential applications to other temporal graph problems.

Multicolour size-Ramsey number for paths

Presenter: Csongor Beke – University of Cambridge
Date: Friday 17th April **Time:** 15:05

The r-colour size-Ramsey number of the path P_k is the smallest m such that some m–edge graph G has a monochromatic P_k in every r-colouring of its edges. The linearity in k has been established by Beck in 1983, but finding the optimal r-dependence has been open since. The lower bound of cr^2k by Krivelevich was suggested to be optimal, while the best upper bound of Dudek and Pralat is a factor of $\log r$ away. In this talk we introduce a new colouring technique that, somewhat surprisingly, yields a $\log r$ improvement in the lower bound, solving the problem up to constants.

This is joint work with Anqi Li and Julian Sahasrabudhe.

Zero Sum Ramsey numbers are linear on d-degenerate Graphs

Presenter: Andrey Shapiro – Kings College London
Date: Friday 17th April **Time:** 15:30

Let p be a prime number and let G be a graph on n vertices and m edges. The zero-sum Ramsey number of G over $\mathbb{Z}_p$, denoted by $R(G, \mathbb{Z}_p)$, is the minimum $\ell \in \mathbb{N}$ such that for any edge-coloring $c : E(K_\ell) \to \mathbb{Z}_p$, there is a subgraph $G' \subset K_\ell$ isomorphic to G and satisfying $\sum_{e \in E(G')} c(e) = 0$.

We prove that if G is a d-degenerate graph, then $R(G, \mathbb{Z}_p) \leq n + (3 + 3d)p$ so long as $m \geq 2pd(d + 1)^2$, p divides m, and $2d < p$. This generalizes a similar result by Colucci and D'Emidio on forests.

Decay-Ramsey Sequences

Presenter: Pedram Noohi – London School of Economics
Date: Friday 17th April **Time:** 15:55

Let $\mathbf{a} = (a_i)_i$ be a non-increasing sequence of non-negative real numbers. We say that a colouring $c : E(K_n) \to \mathbb{N}$ is $\mathbf{a}$-bounded if for all $i \in \mathbb{N}$, that colour i is used at most $\lfloor a_i \binom{n}{2} \rfloor$ times. A sequence $\mathbf{a}$ is said to be *weakly* Decay-Ramsey, if for all $C > 0$ and $k \in \mathbb{N}$ there exists some n such that for all $C\mathbf{a_n}$-bounded colorings there exists a monochromatic K_k, and *strongly* Decay-Ramsey if for all $C > 0$ and $k \in \mathbb{N}$ there exists n_0 such that for all $n \geq n_0$ in every $C\mathbf{a_n}$-bounded there exists a monochromatic K_k. We show that all sequences with a divergent sum are not strongly Decay-Ramsey, and that if a sequence $(a_i)_i$ has a divergent series, then it can also be weakly Decay-Ramsey. Additionally we show that for every divergent positive sequence $(b_i)_i$, there exists a sequence $(a_i)_i$ that is strongly Decay-Ramsey and satisfies $\sum_{i=1}^{\infty} a_i b_i = \infty$.